***Note:*** *Each question carries 10 marks.* *Only hand-written assignment is accepted. Marks will be deducted if student's registration id, name, and section title are not written.*

***Sets***

**Problem 1**

1. Write each of the following sets in set-builder notation.
2. Find the following cardinalities.
   1. = 3
   2. = 9
   3. = 1
3. Write the power set of the following sets:

Solution:

1. Suppose and Find:

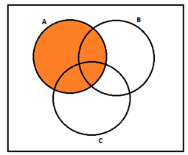
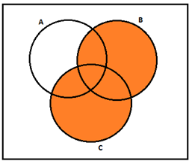
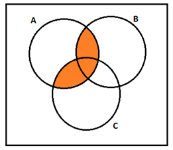
Solution:

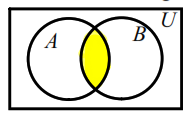
1. Let and have universal set . Then find the following three operations:

Solution:

1. Draw Venn diagrams for and . Based on your drawings, do you think ?

Solution:

A =  =  ; = 

=

1. List all subsets of . How many do you get? List all subsets of , containing but NOT containing .

Solution:

**Problem 2**

1. We form the union of a set with 5 elements and a set with 9 elements. Which of the following numbers can we get as the cardinality of the union: 4, 6, 9, 10, 14, 20?
2. We form the union of two sets. We know that one of them has *n* elements and the other has *m* elements. What can we infer about the cardinality of their union?

The cardinality of their union can be at least equal to the cardinality of the greater set; this happens when one of the set is a subset of the other set, and it can be at most equal to the sum of the cardinality of both the sets, when there is no common element between the sets.

Let and , then

1. We form the intersection of two sets. We know that one of them has *n* elements and the other has *m* elements. What can we infer about the cardinality of their intersection?

The cardinality of the intersection can be at least zero, when there is no common element between the sets and it can be equal to the cardinality of the smaller set when one (smaller) of the set is the subset of the other set.

Let and , then

1. What is the intersection of

i) the sets and ;

ii) the set of girls in this class and the set of boys in this class; null or empty set

iii) the set of prime numbers and the set of even numbers?

**Problem 3**

1. What is the symmetric difference of the set of nonnegative integers and the set of odd integers (contains both negative and positive odd integers).

Solution:

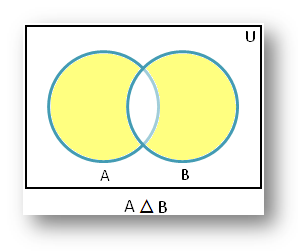
1. Let be the symmetric difference of and (that is ). Now, form the symmetric difference of *A* and *C*. What did you get? Give a proof of the answer using Venn diagram.

Solution:

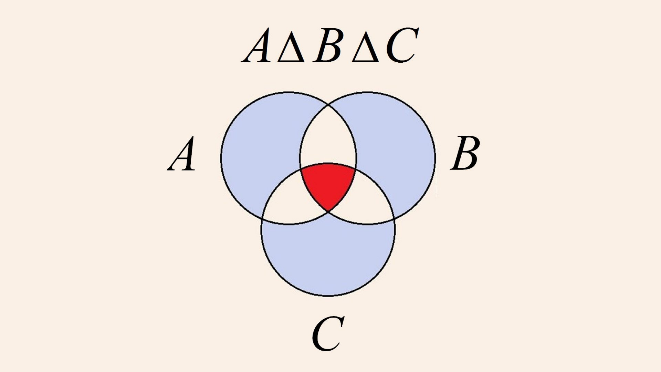
The symmetric difference using Venn diagram of two subsets A and B is a sub set of U, denoted by A △ B and is defined by . Let A and B are two sets. The symmetric difference of two sets A and B is the set and is denoted by . Thus,

or,

Following diagram shows symmetric difference using Venn diagram.



As the given question is meant to be an associative property over symmetric difference, similar diagram can be re-structured as follows:



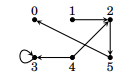
***Relations and Functions***

**Problem 4**

1. Let . Write out the relation R that expresses | (divides) on A. Then illustrate it with a directed graph.

A directed graph can be made accordingly.

1. The following diagram represents a relation R on a set A. Write the sets A and R.



Solution: and

1. Consider the relation on set . Say whether R is reflexive, symmetric and transitive. Provide reason for your answer

Solution: This is reflexive and symmetric relation.

1. Check whether the relation | (divides) on the set Z is equivalence relation?

Solution:

An **equivalence relation** is a relationship on a set, generally denoted by , that is reflexive, symmetric, and transitive for everything in the set.

* (Reflexivity) a ∼ a, [here, a | (divides a such as 5 divides 5] (holds true)
* (Symmetry) if a ∼ b then b ∼ a, [here, a | b then b | a such as 5 | 2 then 2 | 5] (does not hold)
* (Transitivity) if a ∼ b and b ∼ c then a ∼ c. [here, a | b and b | c then a | c, 4 | 8 and 8 | 16 then 4 | 16] (holds true)

Thus, the relation | (divides) on the set Z is not an equivalence relation.

1. Suppose and .

Then is a relation from A to B. Notice that we have . The relation R is the familiar relation ∈ for the set A, that is means exactly the same thing as x ∈ X. Draw an arrow diagram as well as a matrix for the above relation.

Solution:

0/1 Matrix:

**Problem 5**

1. Suppose is defined as . State the domain, codomain and range of f. Find f (10).

Solution:

From the definition it is easy to say that is the domain and codomain of the function *f*. In order to answer for the range of *f*, consider an element y of codomain . We are interested in determining x in the domain such that i.e., . Thus . Since the domain is , y should be such that is an integer. Therefore, the range of f is is an integer}.

1. Consider the set . Is this a function from Z to Z? Explain.

Solution:

Thus

Therefore, is it a function defined for all Z inputs? No. *f* not defined for all domain elements. For example, is not defined.

1. A function is defined as . Verify whether this function is injective and whether it is surjective.

Solution:

is injective

is not surjective because is an odd number for any . Therefore, even integers of the codomain can never be an image of *f*.

1. Consider the cosine function . Decide whether this function is injective and whether it is surjective.

Solution:

**Injectivity**: We know that and . So, different elements in R may have the same image. Hence, f is not an injection.

**Surjectivity**: Since the values of lie between −1 and 1, it follows that the range of is not equal to its co-domain. So, f is not a surjection.

1. Determine whether the relation R on the set of all real numbers is reflexive, symmetric, and/or transitive, where (x, y) ∈ R if and only if

Solution:

1. Not reflexive because (; not symmetric because (); and not transitive ()
2. Not reflexive because ; Symmetric because ; not transitive because
3. Use Venn diagram to illustrate the relationships
4. Determine whether each of these functions from Z to Z is one-to-one. Or not Explain your answer with prove.

Solution:

***Graphs***

**Problem 6**

A friendship graph is given below.

1. Make a array to show who is friend of whom.

(All rows are labelled A, B, C, D and all columns are labelled A, B, C, D. Mark the corresponding entry if they are friends, and cross if they are not).

Solution:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | A | B | C | D | E |
| A |  |  |  |  |  |
| B |  |  |  |  |  |
| C |  |  |  |  |  |
| D |  |  |  |  |  |
| E |  |  |  |  |  |

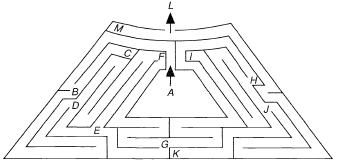
1. Represent the friendship relation, from the graph below, as sets (e.g. , all friends of A. Similarly do for others).



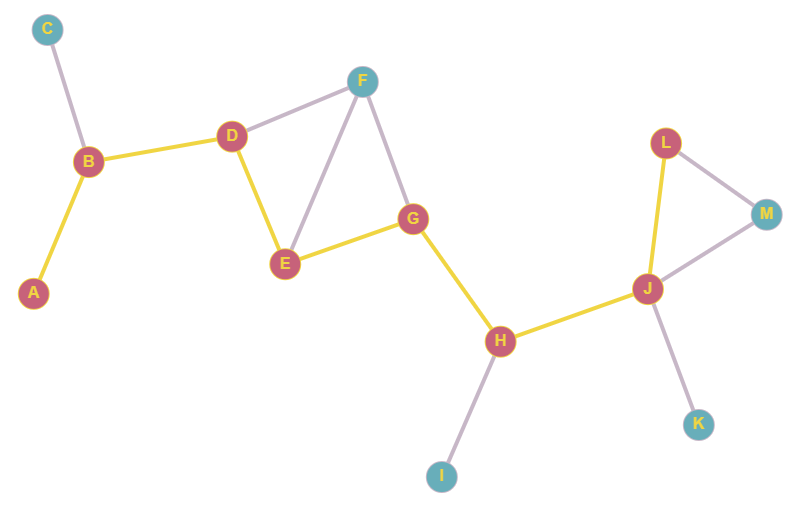
Solution:

**Problem 07**

1. Draw an Influence Graph for the board members of a company if the President can influence the Director of Research and Development, the Director of Marketing, and the Director of Operations; the Director of Research and Development can influence the Director of Operations; the Director of Marketing can influence the Director of Operations; and no one can influence, or be influenced by the Chief Financial Officer.
2. Draw graph of the following maze. You make an edge whenever one labelled corridor is connected to another one.



Solution



1. In a round-robin tournament, the Tigers beat the Blue Jays, the Tigers beat the Cardinals, the Tigers beat the Orioles; the Blue Jays beat the Cardinals, the Blue Jays beat the Orioles, and the Cardinals beat the Orioles. Draw a graph for this outcome.

Solution:

Set of Nodes = {Tigers, Blue Jays, Cardinals, Orioles}

There is a directed graph for this problem.